8. PROBABILITY

Quick Review

1. An experiment which can be repeated any number of times under essentially identical conditions and which is associated with a set of known results, is called a ***random experiment*** or ***trial*** if the result of any single repetition of experiment is not certain and is any one of the associated set.
2. The result of any single repetition of a random experiment is called an ***elementary event*** or ***simple event.***
3. Elementary events are said to be equally likely if no event has preference over other events.
4. A combination of one or more elementary events in a trial is called an ***event.***
5. The favourable cases to a particular event of an experiment are called successes and the remaining cases are called failures with respect to that event.
6. If there are n exhaustive equally likely elementary events in a trial and m of them are favourable to an event A, then m/n is called the ***probability*** of A. It is denoted by P(A).
7. If a trial is conducted n times and m of them are favourable to an event A, then m/n is called relative frequency of A and is denoted by R(A). If $\underset{n\to\infty}{Lt} R(A)$ exists, then the limit is called ***probability*** of A.
8. The set of all possible outcomes (results) in a trial is called ***sample space*** for the trial. It is denoted by S. The elements of S are called ***sample points.***
9. Let S be a sample space of a random experiment. Every subset of S is called an ***event.***
10. Let S be a sample space. The event Ø is called ***impossible event*** and the event S is called ***certain event*** in S.
11. Two events A, B in a sample space S are said to be ***disjoint*** or ***mutually exclusive*** if $A \cap B = \emptyset$.
12. The events $A_1, A_2, \ldots, A_n$ in a sample space S are said to be ***mutually exclusive*** or ***pairwise disjoint*** if every pair of the events $A_1, A_2, \ldots, A_n$ are disjoint.
13. Two events A, B in a sample space S are said to be ***exhaustive*** if $A \cup B = S$.
14. The events $A_1, A_2, \ldots, A_n$ in a sample space S are said to be ***exhaustive*** if $A_1 \cup A_2 \cup \ldots \cup A_n = S$.
15. Two events A, B in a sample space S are said to be ***complementary*** if $A \cup B = S$, $A \cap B = \emptyset$.
16. Let A be an event in a sample space S. An event B in S is said to be ***complement*** of A if A, B are complementary in S. The complement B of A is denoted by $\overline{A}$.
17. The complement of an event A in a sample space S is unique. If $\overline{A}$ is the complement of A then $A \cup \overline{A} = S$, $A \cap \overline{A} = \emptyset$ and $\overline{(\overline{A})} = A$.
18. Let S be a finite sample space. A real valued function $P : P(S) \to R$ is said to be a ***probability function*** on S if (i) $P(A) \geq 0, \forall A \in P(S)$ (ii) $P(S)=1$ (iii) $A, B \in P(S), A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A)+P(B)$.
19. Let S be a finite sample space and P be a probability function on S. If A is an event in S then P(A), the image of A, is called ***probability*** of A.
20. If $A_1, A_2, \ldots, A_n$ are n mutually exclusive events in a sample space S, then
$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$.
21. $P(\emptyset) = 0$
22. If A is an event in a sample space S, then $P(\overline{A}) = 1 - P(A)$.
23. Let A, B be two events in a sample space S. If $A \subseteq B$, then $P(A) \leq P(B)$.
24. If A is an event in a sample space S, then $0 \leq P(A) \leq 1$.

25. Let S be a sample space containing n sample points. If E is an elementary event in S, then P(E) = 1/n.
26. Let S be a sample space containing n sample points. If A is an event in S containing m sample points, then P(A) = m/n.
27. If A is an event in a sample space S, then P(A) = $\frac{n(A)}{n(S)}$, where n(A) is the number of sample points in A and n(S) is the number of sample points in S.
28. If A is an event in a sample space S, then the ratio P(A) : P($\overline{A}$) is called the ***odds in favour*** to A and P($\overline{A}$) : P(A) is called the ***odds against*** to A.
29. **Addition theorem on probability :** If A, B are two events in a sample space S, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
30. If A, B are two events in a sample space S, then $P(B - A) = P(B) - P(A \cap B)$ and $P(A - B) = P(A) - P(A \cap B)$.
31. If A, B, C are three events in a sample space S, then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.
32. If A, B are two events in a sample space, then the event of happening of B after the event A happening is called ***conditional event***. It is denoted by B | A.
33. If A, B are two events in a sample space S and $P(A) \neq 0$, then the probability of B after the event A has occurred is called ***conditional probability*** of B given A. It is denoted by P(B | A).
34. If A, B are two events in a sample space S such that $P(A) \neq 0$, then $P(B \mid A) = \frac{n(A \cap B)}{n(A)}$.
35. **Multiplication theorem of probability :** Let A, B be two events in a sample space S such that $P(A) \neq 0$, $P(B) \neq 0$. Then

    i) $P(A \cap B) = P(A)\, P(B \mid A)$ ii) $P(A \cap B) = P(B)\, P(A \mid B)$
36. Two events A, B in a sample space S are said to be ***independent*** if P(B | A) = P(B).
37. Two events A, B in a sample space S are independent iff $P(A \cap B) = P(A)\, P(B)$.
38. The events $A_1, A_2, \ldots, A_n$ are independent iff $P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1)\, P(A_2) \ldots P(A_n)$.
39. If A, B are two independent events in a sample space S, then (i) $\overline{A}$, B are independent (ii) A, $\overline{B}$ are independent, (iii) $\overline{A}, \overline{B}$ are independent.
40. If $A_1, A_2, \ldots, A_n$ are independent events in a sample space S, then $\overline{A}_1, \overline{A}_2, \ldots, \overline{A}_n$ are also independent events.
41. If $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive events in a sample space S such that $P(A_i) > 0$ for i = 1, 2, …, n and E is any event, then $P(E) = \sum_{i=1}^{n} P(A_i) P(E \mid A_i)$.
42. If A_1, A_2 are two mutually exclusive and exhaustive events and E is any event then

    $P(E) = P(A_1)\, P(E \mid A_1) + P(A_2)\, P(E \mid A_2)$.
43. **Baye's theorem :** If $A_1, A_2, \ldots, A_n$ are mutually exclusive and exhaustive events in a sample space S such that $P(A_i) > 0$ for i = 1, 2, …, n and E is any event with P(E) > 0, then

    $P(A_k \mid E) = \frac{P(A_k)\, P(E \mid A_k)}{\sum_{i=1}^{n} P(A_i)\, P(E \mid A_i)}$ for K = 1, 2, …, n.

44. When two dice are thrown the number of ways of getting a total r is (i) r – 1, if $2 \le r \le 7$ (ii) 13 – r, if $8 \le r \le 12$.

45. When three dice are thrown the number of ways of getting a total r is 27, if r = 10 or 11.

46. If n letters are put at random in the n addressed envelopes, the probability that

    (i) all the letters are in right envelopes = 1/n!

    (ii) atleast one letter may be in wrongly addressed envelope = 1 – 1/n!

47. If A and B are two finite sets and if a mapping is selected at random from the set of all mappings from A into B, then the probability that the mapping is a

    (i) one one function is $\frac{{}^{n(B)}P_{n(A)}}{n(B)^{n(A)}}$.

    (ii) constant function is $\frac{n(B)}{n(B)^{n(A)}}$.

    (iii) one one onto function is $\frac{n(A)!}{n(B)^{n(A)}}$, if n(A)=n(B).

48. Out of n pairs of shoes, if r shoes are selected at random, then the probability that

    i) there is no pair is $\frac{{}^{n}C_{r}\, 2^{r}}{{}^{2n}C_{r}}$.

    ii) there is atleast one pair is $1 - \frac{{}^{n}C_{r}\, 2^{r}}{{}^{2n}C_{r}}$.

49. If p, q are the probabilities of success, failure of a game in which A, B play then

    i) probability A's win = $\frac{p}{1-q^2}$

    ii) probability of B's win = $\frac{qp}{1-q^2}$.

50. If p, q are the probabilities of success, failure of a game in which A, B, C play then

    i) probability of A's win = $\frac{p}{1-q^3}$

    ii) probability of B's win = $\frac{qp}{1-q^3}$

    iii) probability of C's win = $\frac{q^2p}{1-q^3}$